**How does stochastic volatility affect early exercise?**

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We often learn that early exercise of an American put option can be optimal. This decision depends on the stock price, intrinsic value, and continuation value. By using methods like Least Squares Monte Carlo (LSM) or the Binomial Model, we can estimate when early exercise is beneficial. However, both assume constant volatility, which, based on my lessons at the university, rarely holds true in real stock data. To address this, I will introduce stochastic volatility and examine how it influences early exercise decisions. This leads to the key question:

*How does stochastic volatility affect early exercise?*

To answer this, I will use the LSM method introduced by Longstaff and Schwartz (2001)[1] and incorporate Heston’s model (1993)[2] or a similar stochastic volatility process. Under the Heston framework, the stock price follows:

where volatility is not constant but follows another stochastic process. Should I continue with the Heston model, the variance will follow:

This captures volatility clustering and mean reversion, which makes the model more realistic. By simulating these price patterns, I can evaluate how early exercise decisions change under different volatility conditions.

In the LSM approach, continuation value is estimated through least squares regression. Longstaff  
and Schwartz (2001) used only stock price as a regressor, but I propose also including volatility. The continuation value is then estimated as:

This way, we account for how both stock price and volatility impact option value. The rule for early exercise remains:

where the intrinsic value must exceed the estimated continuation value. Identifying these boundaries will show how stochastic volatility influences the likelihood and timing of early exercise.

For testing the model, I am still considering the best approach. Right now, the most logical comparison seems to be with the binomial model to check if early exercise behavior under a constant volatility assumption is different from a stochastic volatility framework. However, this is still a work in progress.

**References**

[1] Longstaff, F. A., & Schwartz, E. S. (2001). Valuing American options by simulation: A simple least squares approach. The Review of Financial Studies, 14 (1), 113-147

[2] Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. The Review of Financial Studies, 6 (2), 327-343